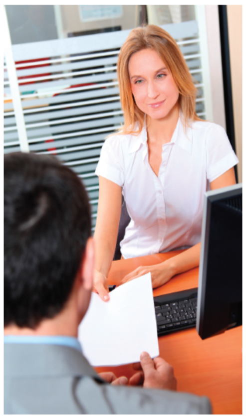
10 Getting Started



When a Loan Is an Investment

Doris works as a personal loan manager at a bank. It is her job to decide whether the bank should lend money to a customer. When she approves a loan, she thinks of it as the bank making an investment in the person who is borrowing the money. Doris is considering a loan application from Leandro, who wants to borrow $10 000 to renovate his garage so that he can use it as a workshop. She expects the money borrowed plus interest to be repaid as a single payment at the end of 2 years. She is considering the following three loan options for Leandro:

Option A: A loan at 6% simple interest

Option B: A loan at 5.5% compound interest with annual compounding

Option C: A loan at 5% compound interest with semi-annual compounding

Which is best for Leandro ?

Option A

time

$$A = P(1 + rt)$$

principal rate

$$A = 10\,000(1 + (0.06)(2))$$

$$= 10\,000(1 + 0.12)$$

$$= 10\,000(1.12)$$

$$= \$11\,200$$

Option B:

$$A = P(1+i)^n$$

$$A = 10\,000(1 + 0.055)^2$$

$$A = 10\,000(1.055)^2$$

$$A = \$11\,130.25$$

Option C:

$$A = P(1+i)^n$$

$$A = 10\,000\left(1 + \frac{0.05}{2}\right)^2$$

$$A = 10\,000(1 + 0.025)^2$$

$$A = 10\,000(1.025)^2$$

$$A = \$10\,506.25$$

Recall from Unit 6 that simple interest is "linear" in nature and involves the formula $A=P(1+rt)$ where A is the sum of the principal and accumulated interest, P is the principal, r is the interest rate per annum, and t represents the time in years.

Option A (6% simple interest): $A = P(1+rt) = 10\,000(1+0.06\times 2) = 10\,000(1.12) =$ \$11 200

PRINCIPAL INTEREST RATE PER ANNUM TIME IN YEARS

Also recall from Unit 6 that compound interest is "exponential" in nature and involves the formula $A = P(1+i)^n$ where A is the future value, P is the principal, i represents the interest rate per compounding period (as a decimal), and n represents the number of compunding periods.

Option B (5.5% compound interest, annually):

$$A = P(1+i)^n = 10\,000(1+0.055)^2 = 10\,000(1.055)^2 = \$11\ 130.25$$

COMPOUNDING PERIODS

PRINCIPAL

INTEREST RATE WAS ANNUAL

Option C (5% compound interest, semi-annually):

$$A = P(1+i)^n = 10\,000\left(1+\frac{0.05}{2}\right)^4 = 10\,000(1+0.025)^4 = 10\,000(1.025)^4 = \$11\ 038.13$$

PRINCIPAL

INTEREST RATE WAS SEMI-ANNUAL (i.e. 0.05/2)

COMPOUNDING PERIODS (i.e. 2 years semi-annually means 4 compounding periods)

It would seem that "Option C" is the best of these three options for Leandro !

Next

The loan manager throws a 4th option out to Leandro :

Option D: A loan at 5% interest, compounded semi-annually, with payments of $2658.18 at the end of every 6-month period for 2 years.

NOTE : This is very different than the previous options. i.e. there is not a single payment at the end of the 2 year period, but rather regular payments over the 2 year period. You can use a "table" to show the repayment of the loan.

TABLE: (fill it in based on the information given):

Why is this $\frac{0.05}{2}$?

Payment Period	Payment ($)	Interest Paid ($) $Balance \times \left(\frac{0.05}{2}\right)$	Principal Paid ($) $Payment - Interest$	Balance ($) $Balance - Principal\ Paid$
0				10 000.00
1	2658.18	10 000 × 0.025 $250	2408.18	7591.82
2	2658.18	7591.82×0.025 $189.80	2468.38	5123.44
3	2658.18	5123.44×0.025 $128.09	2530.09	2593.35
4	2658.18	$64.83	2593.35	$0
Total	$\boxed{10632.72}$	632.72	10 000	

Completed Table

Payment Period	Payment ($)	Interest Paid ($) $Balance \times \left(\frac{0.05}{2}\right)$	Principal Paid ($) $Payment - Interest$	Balance ($) $Balance - Principal\ Paid$
0				10 000.00
1	2658.18	250.00	2408.18	7591.82
2	2658.18	189.80	2468.38	5123.44
3	2658.18	128.09	2530.09	2593.35
4	2658.18	64.83	2593.35	0.00
Total	10 632.72	632.72	10 000	

Next

In summary, the four options that Leandro had for borrowing the $10 000 :

OPTION A: A loan at 6% simple interest would have him pay back a total of $11 200 of which $1200 would be interest.

OPTION B: A loan at 5.5% compound interest with annual compounding would have him pay back a total of $11 130.25 of which $1130.25 would be interest.

OPTION C: A loan at 5% compound interest with semi-annual compounding would have him pay back a total of $11 038.13 of which $1038.13 would be interest.

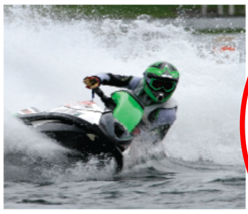
OPTION D: A loan at 5% interest, compounded semi-annually, with payments of $2658.18 at the end of every 6-month period for 2 years would have him pay back $10 632.72 of which $632.72 would be interest

OPTION D is obviously the best option for Leandro while OPTION A is obviously best for the bank!!

10.1 Analyzing Loans

INVESTIGATE the Math

Lars borrowed $12 000 from a bank at 5%, compounded monthly, to buy a new personal watercraft. The bank will use the watercraft as **collateral** for the loan. Lars negotiated regular loan payments of $350 at the end of each month until the loan is paid off. Lars set up an **amortization table** to follow the progress of his loan.



Lars's Amortization Table

Payment Period (month)	Payment ($)	Interest Paid ($) $\left[\text{Balance} \cdot \left(\frac{0.05}{12}\right)\right]$	Principal Paid ($) [Payment − Interest]	Balance ($)
0				12 000.00
1	350	50.00	300.00	11 700.00
2	350	48.75	301.25	11 398.75

A. Complete Lars's amortization table for the first year.

B. At the end of the first year,

- **i)** how much has Lars paid altogether in loan payments?
- **ii)** how much interest has he paid altogether?
- **iii)** how much of the principal has he paid back?

C. At the end of the first year, what is the balance of Lars's loan?

Payment Period (month)	**Payment ($)**	**Interest Paid ($)** $Balance \times \left(\frac{0.05}{12}\right)$	**Principal Paid ($)** $Payment - Interest$	**Balance ($)** $Balance - Principal\ Paid$
0		0.0041666		12 000.00
1	350	50.00	300.00	11 700.00
2	350	48.75	301.25	11 398.75
3	350			
4	350			
5	350			
6	350			$10 181.14
7	350			
8	350			
9	350			
10	350			
11	350			
12	350			
Total				

Completed Table

Payment Period (month)	Payment ($)	Interest Paid ($) $Balance \times \left(\frac{0.05}{12}\right)$	Principal Paid ($) $Payment - Interest$	Balance ($) $Balance - Principal\ Paid$
0				12 000.00
1	350	50.00	300.00	11 700.00
2	350	48.75	301.25	11 398.75
3	350	47.49	302.51	11 096.24
4	350	46.23	303.77	10 792.47
5	350	44.97	305.03	10 487.44
6	350	43.70	306.30	10 181.14
7	350	42.42	307.58	9873.56
8	350	41.14	308.86	9564.70
9	350	39.85	310.15	9254.55
10	350	38.56	311.44	8943.11
11	350	37.26	312.74	8630.37
12	350	35.96	314.04	8316.33
Total	4200.00	516.33	3683.67	

Total of the loan payments after 12 months

Total interest paid after 12 months

Total paid towards principal after 12 months

Balance of the loan after 12 months

Next

b) N = ?
I% = 5
PV = 12 000
PMT = −350
FV = 0
P/Y = 12
C/Y = 12

N = 37.073....
It will take 38 months.

* c)

You can use "technology" (spreadsheet, math app, math software, etc.) to help answer questions throughout this unit. We will be using the TI-83+ (or TI-84) for such purposes:

APPLY the Math

EXAMPLE 1 Solving for the term and total interest of a loan with regular payments

As described on page 636, Lars borrowed $12 000 at 5%, compounded monthly. After 1 year of payments, he still had a balance owing. * $350/month

a) In which month will Lars have at least half of the loan paid off? click
b) How long will it take Lars to pay off the loan? click
c) How much interest will Lars have paid by the time he has paid off the loan? click

N = number of payments
I% = interest rate (actual %)
PV = present value (principal)
PMT = payment amount (negative)
FV = future value (what is left to pay) (negative)
P/Y = payments per year
C/Y = compounding periods per year

N =
I = 5
PV = 12000
PMT = −350
FV = 6000
P/Y = 12
C/Y = 12

N = 19.25 ⇒ 20th month

EXAMPLE 2 Solving for the future value of a loan with a single loan payment

Trina's employer loaned her \$10 000 at a fixed interest rate of 6%, compounded annually, to pay for college tuition and textbooks. The loan is to be repaid in a single payment on the maturity date, which is at the end of 5 years.

a) How much will Trina need to pay her employer on the maturity date? What is the accumulated interest on the loan?

For this example, we will focus on part "a" since the "graphing nature" of parts b & c are not relevant here with respect to the curriculum guide/outcomes:

$$\left.\begin{aligned} A &= P(1+i)^n \\ n &= 5 \\ i &= 0.06 \\ P &= 10\,000 \end{aligned}\right\}$$

$A = 10\,000(1+0.06)^5$ = \$ 13 382.26 is what she needs to pay back.

Therefore, accumulated interest is \$3382.26

click

$$A = P(1+i)^n$$

$$A = 10\,000(1+0.06)^5$$

$$= 10\,000(1.06)^5$$

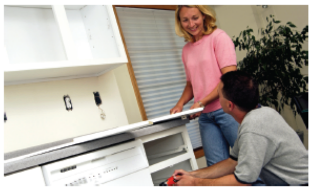
$$\doteq \$13\,382.26$$

$$\text{Interest} = 13\,382.26 - 10\,000$$

$$\boxed{= \$3\,382.26}$$

EXAMPLE 3 Solving for the present value and interest of a loan with a single payment

Annette wants a home improvement loan to renovate her kitchen. Her bank will charge her 3.6%, compounded quarterly. She already has a 10-year GIC that will mature in 5 years. When her GIC reaches maturity, Annette wants to use the money to repay the home improvement loan with one payment. She wants the amount of the payment to be no more than $20 000.



a) How much can she borrow?
b) How much interest will she pay?

Paper/Pencil (formula):

$A = P(1+i)^n$

$A = 20\,000$

$n = 20$ (i.e. 5 yrs, compounded quarterly)

$i = \frac{0.036}{4} = 0.009$ (i.e. 3.6%, compounded quarterly)

$20\,000 = P(1+0.009)^{20}$

$P = \frac{20\,000}{(1.009)^{20}}$

$P = \frac{20\,000}{1.1962538}$

$P = 16\,718.86$

She can borrow $16 718.86

She will pay 20 000-1618.86

or $3281.14 in interest

Using technology:

click

Loan:

$A = P(1+i)^n$

$20\,000 = P\left(1+\frac{0.036}{4}\right)^{5\times 4}$

$20\,000 = P(1+0.009)^{20}$

$\frac{20\,000}{1.009^{20}} = \frac{P(1.009)^{20}}{1.009^{20}}$

$P = \$16\,718.86$

b) $20\,000 - 16\,718.86$

$\boxed{\$3281.14}$

Interest

EXAMPLE 4 Solving for the payment and interest of a loan with regular payments

Jose is negotiating with his bank for a **mortgage** on a house. He has been told that he needs to make a 10% down payment on the purchase price of $225 000. Then the bank will offer a mortgage loan for the balance at 3.75%, compounded semi-annually, with a term of 20 years and with monthly mortgage payments.

a) How much will each payment be?

b) How much interest will Jose end up paying by the time he has paid off the loan, in 20 years?

c) How much will he pay altogether?

click

click

a) 10% of 225 000

= 0.10 × 225 000

= $22 500 (downpayment)

Borrow:

225 000 − 22 500

= $202 500

N = 20×12

I = 3.75

PV = 202 500

PMT $\boxed{?}$

FV = 0

P/Y = 12

C/Y = 2

Payment:

$1197.55

(b) Total Paid

240 × 1197.55

= $287 412

Interest = 287 412

− 202 500

$\boxed{\$84\,912}$

(c)

EXAMPLE 5 Relating payment and compounding frequency to interest charged

Bill has been offered the following two loan options for borrowing $8000. What advice would you give?

Option A: He can borrow at 4.06% interest, compounded annually, and pay off the loan in payments of $1800.05 at the end of each year.

Option B: He can borrow at 4.06% interest, compounded weekly, and pay off the loan in payments of $34.62 at the end of each week.

click

click

PULL

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Example1_page 637 of PoMath12_parta.wmv

Example4_page 643 of PoMath12_partbandc.wmv

Example1_page 637 of PoMath12_partb.wmv

Example4_page 643 of PoMath12_parta.wmv

Example5_page 645 of PoMath12_optA.wmv

Example5_page 645 of PoMath12_optB.wmv

Example3_page 642 of PoMath12_partaandb.wmv

Example2_page 639 of PoMath12_parta.wmv